<u>SPEED OF SOUND: Kundt's Tube</u> Fall 2014

OBJECTIVE:

The goal of this experiment is to measure the speed of sound, $\boldsymbol{v_s}$, in the air. In the first method the speed is found by measuring several resonant frequencies (part 1) and wavelengths for several resonant frequencies (part 2) (both are usual methods for Kundt's tube). In the second method we will use white noise (generated by a computer, Lab View) to find resonant frequencies formed within a resonant tube, using fast Fourier transform (FFT) method, which is the best known and widely used method of transporting time domain data (pulse of the microphone) into the frequency domain (series of the resonant frequencies present in the tube). The accepted value for the speed of sound in ideal gas (like air, at normal temperature and pressure) is: $\boldsymbol{v = v_0\sqrt{1+\alpha T}}$ where $\boldsymbol{v_0} = 331.5\,\text{m/s}$ and $\boldsymbol{\alpha = \frac{1}{273.2}}\,(^o\text{C})^{-1}$ is the coefficient of thermal expansion for ideal gas and $\boldsymbol{T}$ is the temperature of gas in $\boldsymbol{^o\text{C}}$.

BACKGROUND:

Reflection of sound; standing waves

In a cylindrical tube filled with air or some other gas, the sound wave is approximately plane wave. Its amplitude is approximately constant. The wave reflects at the end of the tube. If the tube is closed with a hard wall, the wave reflects, but with the changed sign: the reflected wave has a length difference of $\boldsymbol{\lambda}/2$ with respect to the incoming wave, that is there is a difference in phase by $\boldsymbol{\pi}$. Incoming and reflected wave interfere to produce <u>*standing wave*</u>, which has phase speed equal to zero (wave does not move, or positions of nodes and antinodes do not move), which means that all the points along the wave have the same phase.

For the closed tube (closed on both sides, like ours), let's assume that the source of sound waves is at point $\boldsymbol{S}$ (fig. 1) and the wall, where the reflection occurs, is at $\boldsymbol{W}$. The displacement from equilibrium, $\boldsymbol{u_1(x,t)}$, of point $\boldsymbol{M}$, produced by the incoming sinusoidal wave, at the position that is $\boldsymbol{L - x}$ from the source $\boldsymbol{S}$ is:

$$\boldsymbol{u_1(x,t) = u_m \cos 2\pi\left(\frac{t}{T} - \frac{L-x}{\lambda}\right)}.$$

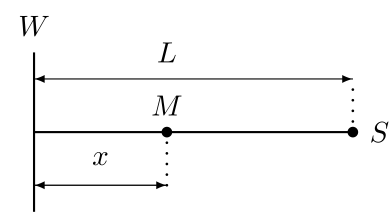


Figure 1. Describing the standing waves.

Displacement at point $\boldsymbol{M}$ due to the reflected wave, because of the phase difference of $\boldsymbol{\pi}$ ($\boldsymbol{\lambda}/2$ in length) and traveled path $\boldsymbol{L+x}$, is:

$$\boldsymbol{u_2(x,t) = u_m \cos 2\pi\left(\frac{t}{T} - \frac{L+x+\frac{\lambda}{2}}{\lambda}\right) = -u_m \cos 2\pi\left(\frac{t}{T} - \frac{L+x}{\lambda}\right)}.$$

Resultant displacement from equilibrium, $\boldsymbol{u(x,t)}$, at point $\boldsymbol{M}$ is:

$$\boldsymbol{u(x,t) = u_1 + u_2 = -2u_m \sin 2\pi\frac{x}{\lambda} \sin 2\pi\left(\frac{t}{T} - \frac{L}{\lambda}\right)}$$

The phase of the oscillations $\boldsymbol{2\pi\left(\frac{t}{T} - \frac{L}{\lambda}\right)}$ does not depend on x, the distance from the wall. The phase is the same for all the points of the resulting wave at the certain moment. It means that **the speed of that phase is equal to zero: we have produced the standing wave**.

The resultant amplitude of the displacement for the particles is: $\boldsymbol{A = -2u_m \sin 2\pi\frac{x}{\lambda}}$

The standing wave amplitude A changes with the distance from the wall, x. At all the points at distances $\boldsymbol{x = k\frac{\lambda}{2}}$ (k = 0, 1, 2, ...) from the wall, the amplitude is equal to zero. These points are called nodes of the standing wave. We see that the node is at the point of the wall, and the distance between the consecutive nodes is $\boldsymbol{\lambda/2}$. At all the points at distances $\boldsymbol{x = (2k+1)\frac{\lambda}{4}}$ the amplitude changes between its extreme values $\boldsymbol{2u_m}$ and $\boldsymbol{-2u_m}$. These points are called antinodes, and the distance between consecutive antinodes is $\boldsymbol{\lambda/2}$.

If the tube is open at one end, so that the reflection occurs on the same gas as inside the tube, there is no change in the phase of π ($\lambda/2$ in length). We get the resultant displacement by summing oscillations:

$$u_1(x,t) = u_m \cos 2\pi \left(\frac{t}{T} - \frac{L-x}{\lambda}\right) \quad \text{and} \quad u_2(x,t) = u_m \cos 2\pi \left(\frac{t}{T} - \frac{L+x}{\lambda}\right).$$

Resultant displacement of particles from equilibrium, $u(x,t)$, at point M is:

$$u(x,t) = u_1 + u_2 = 2u_m \cos 2\pi \frac{x}{\lambda} \cos 2\pi \left(\frac{t}{T} - \frac{L}{\lambda}\right)$$

The resultant amplitude of the displacement for the particles is: $A = 2u_m \cos 2\pi \frac{x}{\lambda}$

Amplitude is equal to zero at points $x = (2k+1)\frac{\lambda}{4}$ ($k = 0, 1, 2, ...$) from the wall, called nodes. At all the points at distances $x = k\frac{\lambda}{2}$ the amplitude changes between its extreme values $2u_m$ and $-2u_m$, called antinodes. Therefore, at the open end of the tube ($x = 0$) there is an antinode for particle displacement.

A standing sound wave has particle displacement nodes (points where the air does not vibrate) and displacement antinodes (points where the amplitude of the air vibration is a maximum). Pressure nodes and antinodes also exist within the standing waveform, 90^o relative to the displacement nodes and antinodes. When the air of the two displacement antinodes are moving toward each other, the pressure of the pressure antinode is maximum. When they are moving apart, the pressure goes to a minimum. Patterns of the first (fundamental mode) and second mode of the pressure standing wave are presented in Fig. 2. In the figure, "S" stands for speaker and "M" stands for microphone.

Appendix B has an animation of a corresponding transverse standing wave with the two waves producing it, moving in the opposite directions (done in Mathematica).

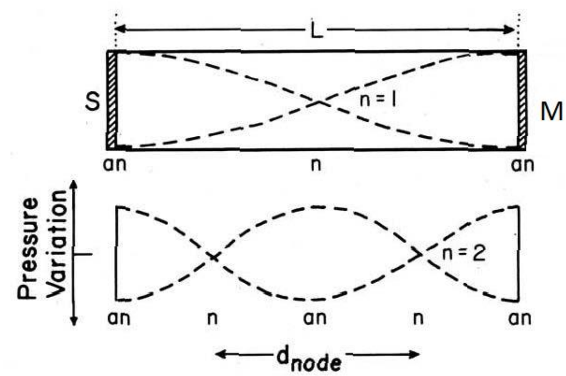


Figure 2. Standing waves in a closed tube of length L. The ends are regions of maximum pressure vibration (antinodes).

Resonating columns, white noise, and FFTs (fast Fourier transforms)

Any length of a tube forms a natural resonant cavity that will preferentially amplify sound waves whose frequency precisely matches those frequencies described by the equation:

$$f_n = \frac{n v_s}{2L} = \frac{v_s}{\lambda_n} \text{ with } n = 1, 2, ... \text{ where } \lambda_n = \frac{2L}{n} \tag{1}$$

where f_n is a resonant frequency, n is an integer (the "overtone number"), v_s is the speed of sound in the cavity, and L is the length of the cavity.

As the equation suggests, there are infinite number of possible resonances for a given length L. If a white-noise source (equal intensity at all frequencies) is introduced to the cavity, the tube will "naturally" select and amplify frequencies that satisfy Eq. (1). At the other end, a microphone "listens" for these resonant frequencies and, using Fourier transform analysis, these frequencies can be measured. Since Eq. (1) gives:

$$v_s = f_n \lambda_n = \frac{f_n 2L}{n} \tag{2}$$

we have a straightforward and elegant way to measure the speed of sound.

NOTE: The following **empirical formulas** give somewhat more accurate description of the resonance requirements for standing waves in a resonant tube.

For a closed-closed or open-open tube: $\lambda_n = \frac{2}{n}(L + 0.8d), n = 1, 2, 3, ...$
where L is the length of the tube and d is the diameter.

For a closed-open tube or tube open on one side: $\lambda_n = \frac{4}{n}(L + 0.4d), n = 1, 3, 5, 7, ...$
where L is the length of the tube and d is the diameter.

NOTE: When using the microphone to investigate the waveform within the tube, be aware that the microphone is a **pressure transducer**. A maximum signal, therefore, indicates a pressure antinode (a displacement node) and a minimum (zero) signal indicates a pressure node (displacement antinode).

General principles of FFTs

(a) *The Fourier series.* The Fourier transform relates a periodic function, $S(t)$ or signal, that can be represented (subject to a few conditions known as the *Dirichlet criteria*) using a Fourier series,

$$S(t) = a_0 + \sum_{k=1}^{\infty} |a_k \cos(2\pi k f_1 t) + b_k \sin(2\pi k f_1 t)| \tag{3}$$

shown in red, in fig. 3, to the function's frequency domain, $\hat{S}$, shown in blue. The component frequencies, spread across the frequency spectrum, are represented as peaks in the frequency domain. Fourier transform measures which and how much, of an individual frequency, is present in a function $S(t)$. The period , T, of the signal equals $1/f_1$, the reciprocal of the fundamental frequency. The other spectral components (or lines) of the signal are overtones occurring with spacing $\Delta f = f_1$ at $f_k = kf_1 \quad (k = 2, 3, ...)$. The Fourier coefficients (a_k, b_k) associated with the spectral lines are the

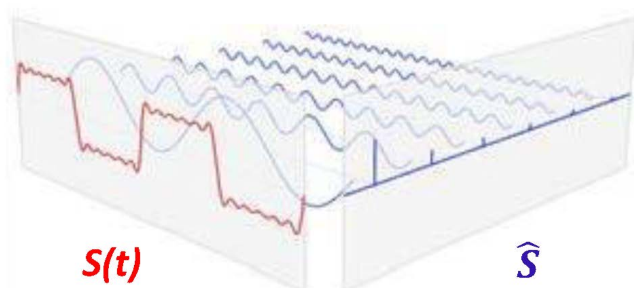


Figure 3.

amplitudes of the various cosine and sine components. The coefficient a_0 is the coefficient associated with the zero-frequency (dc) component. Since $\sin(\theta_1 + \theta_2) = \sin\theta_1 \cos\theta_2 + \cos\theta_1 \sin\theta_2$, eq. 3 can be rewritten as

$$S(t) = a_0 + \sum_{k=1}^{\infty} A_k \sin(2\pi k f_1 t + \delta_k) \tag{4}$$

where δ_k is a phase angle such that $a_k = A_k \sin\delta_k$ and $b_k = A_k \cos\delta_k$ (5)

with $A_k = \sqrt{(a_k^2 + b_k^2)}$. By virtue of orthogonallity relationships Fourier coefficients can be obtained from

$$a_0 = \frac{1}{T}\int_{-T/2}^{T/2} S(t)dt$$

$$a_k = \frac{2}{T}\int_{-T/2}^{T/2} S(t)\cos(2\pi k f_1 t)dt = \frac{2}{T}\int_0^T S(t)\cos(2\pi k f_1 t)dt, \;\; b_k = \frac{2}{T}\int_0^T S(t)\sin(2\pi k f_1 t)dt \tag{6}$$

where $k \leq 1$. One can find A_k and δ_k using equations (5), which permit representation of the original time-dependent signal in the frequency domain. Figures 4a and b show schematically such a transformation for repetitive signal with period T. The first spectral line occurs at frequency $1/T$ and the others with spacing $1/T$. Notice that the spectral lines continue indefinitely.

(b) *Discrete Fourier analyses.* Let examine now the more realistic situation in which discrete data from a nonrepetitive signal is gathered during a finite time interval. When a signal is **sampled** at time intervals Δt using an analogue-to-digital converter (A/D converter), a finite number of

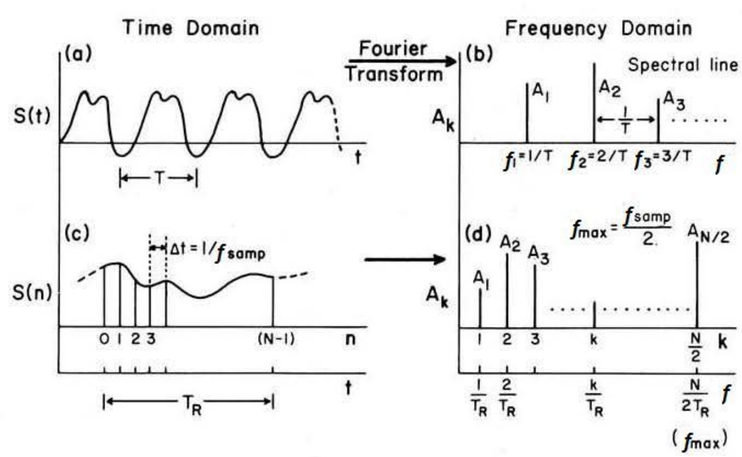


Figure 4. Schematic representation of Fourier transformation of signals from time to frequency domain.

samples(N) is taken during a finite record time ($\boldsymbol{T_R}$) with $\boldsymbol{T_R = N\Delta t}$, fig. 4c. Although in reality we do not know what happens to the signal before or after this observation time, the Fourier transform procedure implicitly assumes that the signal is periodic with period $\boldsymbol{T_R}$. As we have seen from fig. 4a and b, this implied periodic time will control not only the lowest frequency $\boldsymbol{f_1 = 1/T_R}$ but also the spacing, $\boldsymbol{\Delta f = 1/T_R}$, between the discrete frequencies corresponding to the spectral lines. The sampling frequency $\boldsymbol{f_{samp}}$, that is equal to the reciprocal of the time between data samples ($\boldsymbol{\Delta t}$), controls the highest frequency component ($\boldsymbol{f_{max}}$) in the Fourier series, fig.4d. This can be seen as follows. If $\boldsymbol{N}$ equally spaced data samples are gathered in the time domain, then $\boldsymbol{N/2}$ frequencies with equal spacing $\boldsymbol{\Delta f}$ are generated in the frequency domain. Only half as many spectral lines are generated as there are time samples because each frequency actually contains two bits of data - amplitude and phase. Thus $\boldsymbol{f_{max} = (N/2)\Delta f = (N/2)(1/T_R) = f_{samp}/2}$. This maximum frequency is often called the Nyquist frequency ($\boldsymbol{f_{NQ} = f_{max}}$).
In fig. 4c we have a continuous signal that corresponds to a sampling interval of zero, $\boldsymbol{\Delta t = 0}$. Thus $\boldsymbol{f_{samp}}$ (and hence $\boldsymbol{f_{max}}$) are infinite so that the spectral lines of the Fourier transform (FT) in fig. 4b continue indefinitely. Sampling of a continuous signal, as shown in fig. 4c, yields a discrete Fourier transform (DFT), fig. 4d, which is an approximation to the Fourier transform of the continuous signal in that the DFT cannot yield spectral lines above $\boldsymbol{f_{samp}/2}$.

We still have to cast eqs. 6 in forms suitable for digital analysis. Replacing $\boldsymbol{t}$ by $\boldsymbol{n\Delta t, f_1}$ by $\boldsymbol{1/(N\Delta t)}$ ($\boldsymbol{= 1/T_R}$) and $\boldsymbol{dt/T}$ by $\boldsymbol{\Delta t/(N\Delta t)}$ we obtain:

$$a_k = \frac{2}{N}\sum_{n=0}^{N-1} S(n)\cos(2\pi kn/N), \quad b_k = \frac{2}{N}\sum_{n=0}^{N-1} S(n)\sin(2\pi kn/N) \tag{7}$$

where $\boldsymbol{S(n)}$ are the signals obtained at times $\boldsymbol{t(n) = n\Delta t}$. These equasions allow numerical determination of the $\boldsymbol{a_k}$ and $\boldsymbol{b_k}$ coefficients. Equations 5 can then be used to obtain the amplitudes $\boldsymbol{A_k}$ and phases $\boldsymbol{\delta_k}$ associated with the various $\boldsymbol{f_k = k/T_R}$ for $\boldsymbol{k = 1, 2, ..., N/2}$.

The fast Fourier transform (FFT) is simply a procedure for carrying out a DFT efficiently. Most FFT's require $\boldsymbol{N}$ time samples where $\boldsymbol{N}$ equals 2 raised to some integral power, e. g. 512 or 1024.

(c) For a periodic function $\boldsymbol{S(t)}$, like one on the fig. 5, if we wish to include all the frequencies contained in our signal, the sampling frequency should be at least twice the highest frequency in the signal to be analyzed. If this condition is not satisfied the alias frequencies could be generated. In the case when our source contains frequencies that lie above $\boldsymbol{\frac{1}{2} f_{samp}}$, we could use an anti-alias filter, to remove frequencies in this region. Figure 5 shows an example of good sampling. $\boldsymbol{\Delta T}$ is called the **sampling interval**. Then the sampled function is given by the sequence: $\boldsymbol{s(nT)}$, for integer values of $\boldsymbol{n}$. The **sampling frequency** or **sampling rate**, $\boldsymbol{f_s}$, is defined as the number of samples obtained in one second (samples per second), thus $\boldsymbol{f_s = \frac{1}{\Delta T}}$. Reconstructing a continuous function from samples is done by interpolation algorithms.

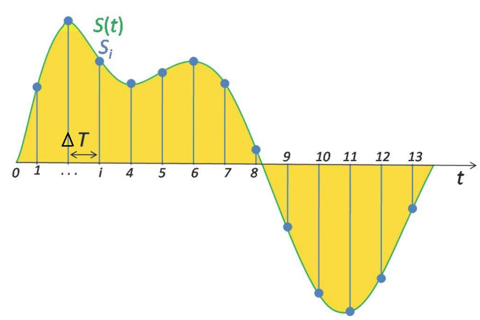
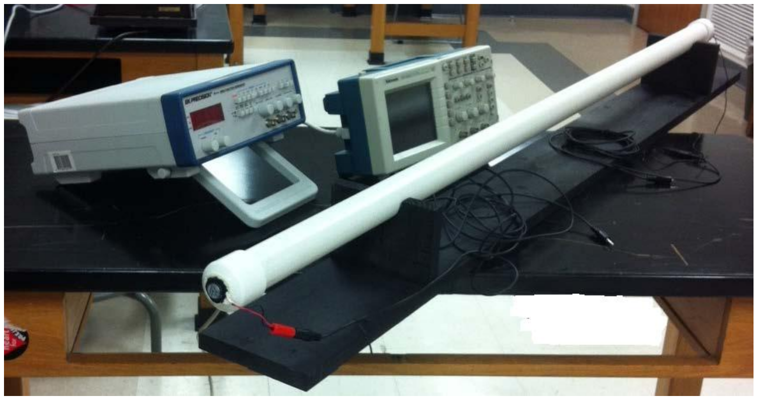


Figure 5. The discrete samples are indicated by the vertical lines.

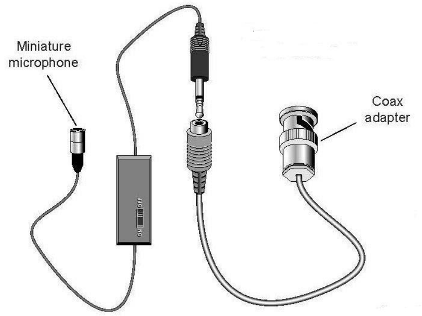
See **Appendix A** for an example of the process described above, in the case of a representation of the white noise signal, using Mathematica.

Sites with some explanations of FFT:
http://betterexplained.com/articles/an-interactive-guide-to-the-fourier-transform/
http://en.wikipedia.org/wiki/Frequency_domain

APPARATUS:



- A plastic (PVC) tube with a wooden stand
- Three PVC end-plugs, one with a built-in speaker, one with a hole for a miniature microphone (with an amplifier) and a long metal rod (for use with the oscilloscope) shown in the figure, and one with a built in microphone (for use with the computer)



- Connectors for direct attachment of the speaker to the function generator or the computer's source of sound or white noise, and connectors for direct attachment of the microphone to the oscilloscope or the computer
- Microphone metal probe rod
- Function generator (BK PRECISION 4011A, 5 MHz)
- Oscilloscope (Tektronix TDS 1012 or better, Two channel digital storage oscilloscope)

EXPERIMENTAL SETUP for the first part:

1. Connect (or check connection) speaker (built in one PVC end-plug) to the function generator. Connect (or check connection) microphone with an amplifier (mounted through a hole of the other PVC end-plug) to the oscilloscope as represented schematically in Fig. 6.

2. Set the frequency of sine-signal from the function generator to approximately 100 Hz, and the output level to zero, then turn it on. Slowly raise the output level until you hear a sound from the speaker.

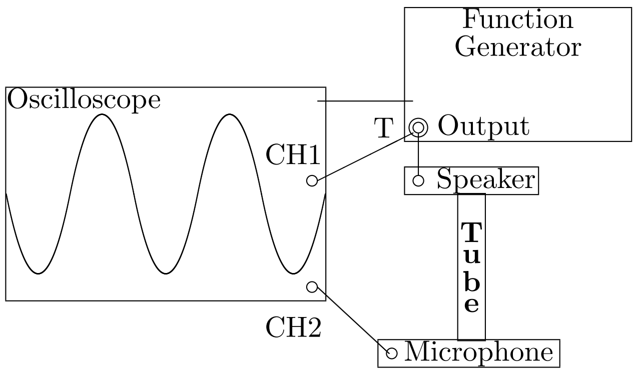


Figure 6. Equipment setup for standing waves.

⇒ **CAUTION:** *You can damage the speaker by overdriving it. Raise the amplitude cautiously. The sound from the speaker should be clearly audible, but not loud. Note also that many function generators become more efficient at higher frequencies, so you may need to reduce the amplitude as you raise the frequency.*

⇒ **NOTE:** *It can be difficult to find resonant frequencies at low frequencies (0-300 Hz). If you have trouble with this, try finding the higher frequency resonant mode first, then use your knowledge of resonance modes in a tube to determine the lower resonant frequencies. Be sure to check to make sure that resonance really occurs at those frequencies.*

3. Turn on the oscilloscope and switch on the microphone's amplifier, set the sweep speed to approximately match the frequency of the signal generator and set the gain until you can clearly see the signal from the microphone. If you can't see the microphone signal, even at maximum gain, adjust the frequency of the signal generator until the sound from the speaker is a maximum. Then raise the amplitude of the signal generator until you can see the signal clearly on the oscilloscope.

4. You can now find resonant frequencies by adjusting the frequency of the sound waves and listening for a maximum sound and/or watching for a maximum signal on the oscilloscope. Find and make a record of **at least 5 resonant frequencies** starting from the fundamental frequency. Plotting $\boldsymbol{n}$'th frequency, $\boldsymbol{f_n}$, on $\boldsymbol{y}$-axis and $\boldsymbol{\dfrac{n}{2(L+0.8d)}}$ on $\boldsymbol{x}$-axis will give speed of sound as a slope.

EXPERIMENTAL SETUP for the second part:

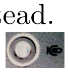
1. Remove the end-plug from the tube. Pull first the microphone and then the rod through a hole in the end-plug. Attach the microphone to the end of the rod with an electrical tape and put the microphone, rod, and the end-plug back on the resonant tube (with mic and rod inside the tube).

2. Make (or leave) the same connections as in the first part.
3. You will use the microphone taped on the rod, to determine the characteristics of the standing waves for at least 5 resonant frequencies you found in the first part. You might have problem with the first two frequencies, so start the measurement with the third. As you move the microphone, taped on the rod, down the length of the tube, note the positions where the oscilloscope signal has a maximum and where it has a minimum. Record these positions in table 1, starting with the position of the rod fully inside the tube. You will not be able to reach the antinode on the speaker because the rod is not long enough, so record as many as you can.

4. Repeat the above procedure for **at least 5 different resonant frequencies** and record your result in the table. Draw the standing wave for the pressure inside the tube, for each frequency, starting with f_1.

EXPERIMENTAL SETUP for the third part (Fourier transform):

1. Remove the speaker male connection to the function generator. Use female-male connecting wire to connect it now to the speaker-outlet  at the front panel of the computer.

2. Remove the end-plug with the microphone with an amplifier and the rod. Instead, attach the other end-plug with a microphone glued to its center. This microphone does not have an amplifier, because the computer sound card will amplify the sound instead. Connect the other end of the microphone cable to the microphone-outlet  at the front panel of the computer, as represented schematically in Fig. 7.

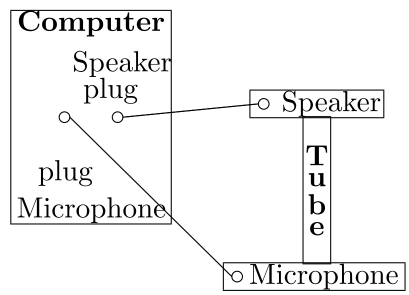
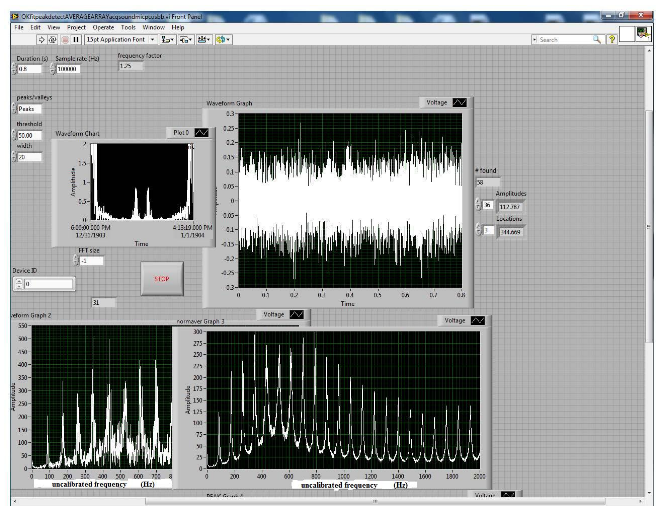


Figure 7. Setup for FFt method.

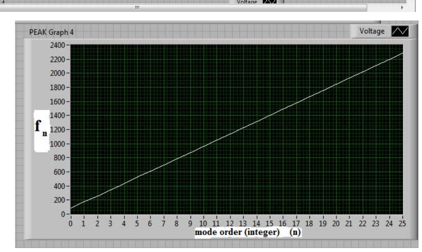
3. Open the Lab View program which will produce white noise (signal with large array of frequencies, named "**DC...**"). You will send that signal to the speaker. To analyze the microphone signal open the program which uses Fourier transform to transform time-signal into the frequency-signal (named "**OK...**").

4. Before starting the Fourier transform program, open on your desktop a new folder with the name you want (for example: sound-output).



5. Turn on the program to produce the signal (click on the **white** arrow) and then run the analysis (click the **white** arrow on "OK"). Run 20-30 cycles of the "**OK**" program. To stop the run, click the "**STOP**" button, that is in the middle between four plots (**NOT** the "stop acquision" button to the right of the starting arrow). A window will appear asking you the location of the file (choose DESKTOP, and the folder you have just opened, and click SAVE. If the small window appear with a message "You don't have permission to acess this folder", just click "CANCEL".

6. Make decision on appropriate threshold and peak-width which fitting routine in "OK" program uses to find the centroids of the peaks representing uncalibrated resonant frequencies. Run again 40-50 cycles. Save the file with another name. Delete file representing the first run, before you have actually decided on the appropriate threshold and peak-width. Save the run when you are sure that the correct peaks representing the uncalibrated frequencies have been saved.



7. Couple of windows will show you the progress of the analysis (file "OK").The top-right graph shows the time-signal microphone receives after the white noise signal was sent to the speaker. After the Fourier transform, the uncalibrated frequencies found in the tube are presented in the low-left graph, and averaged version (average of 10) of the same, in the lower-right graph. Visible are uncalibrated resonant frequencies and they are equidistant, with the uncalibrated fundamental frequency being the difference between each consecutive ones. The last figure shows a plot of the uncalibrated frequencies versus their order, $\boldsymbol{n}$, which has a slope equal to the uncalibrated fundamental frequency. The peaks centroids, representing the uncalibrated resonant frequency positions, were established using Lab View fitting routine.

8. After choosing correct threshold and peak-width, you get an useful output, saved in an file in EXCEL format. Open the file and insert a column before the column with the uncalibrated resonant frequencies (the last one). Fill the inserted coulmn with intehers from 1 to $\boldsymbol{n}$ in increments of 1 (representing mode order). Use the column with the integers and the uncalibrated resonant frequencies for analyses and plotting. You can also prepare a column with actual resonant frequencies first and then start analyses and plotting.

9. In order to get correct value for the resonant frequency from the results of the fit, you should calibrate the low graphs. A certain uncalibrated frequency peak represents the number of samples found during the time of sampling (0.8 second for the run plotted). In order to find actual frequency, you need to find number of samples during one second, which in this specific case means that we have to multiply the peak position (uncalibrated sample frequency from the lower plots) by 1.25 to get the correct measured resonant frequency.

<u>ANALYSIS:</u>

Part 1:

1. Using data from Table 1 plot the resonant frequencies, $\boldsymbol{f_n}$, as function of the inverse of the $\boldsymbol{n}$'th wavelength, $\dfrac{\boldsymbol{n}}{\mathbf{2(\boldsymbol{L}+0.8\boldsymbol{d})}}$ (using empirical relation for wavelength that involves diameter beside the length of the tube). The slope and uncertainty of this plot is the speed of sound in air at room temperature, and its uncertainty.

2. Comment the measured intercept.

Part 2:

1. How does the average node-antinode distance for the $\boldsymbol{n}$'th frequency relate to the $\boldsymbol{n}$'th wavelength?

2. Plot the resonant frequencies, $\boldsymbol{f_n}$, as function of the inverse of the measured $\boldsymbol{n}$'th wavelength, $\dfrac{\mathbf{1}}{\mathbf{4(\boldsymbol{N}-\boldsymbol{A})_{avg,n}}}$. The slope and uncertainty of this plot is the speed of sound in air at room temperature, and its uncertainty.

3. Comment the measured intercept.

4. Compare the results for the speed of sound in parts 1 and 2.

Part 3:

1. Use first 20 or so "resonant frequencies" (from the peak-fitting routine) entered in table 3, or the values for the "resonant frequencies" you have saved as excel file. These "frequencies" represent number of samples in time $\boldsymbol{\Delta t}$= 0.8 seconds, for example. To find the actual resonant frequencies, find number of samples during 1 second, multiplying "resonant frequencies" with $\dfrac{\mathbf{1}}{\boldsymbol{\Delta t}} = \mathbf{1.25}$ here. Plot actual $\boldsymbol{n^{th}}$ resonant frequencies (on y axis) as function of order $\boldsymbol{n}$ (on x axuis). The slope of this plot is a fundamental frequency. Using the fundamental frequency and its uncertainty find the speed of sound and its uncertainty. Use the empirical relation for the wavelength to find the wavelength of the fundamental frequency.

2. Comment the measured intercept of the $\boldsymbol{f_n} - \boldsymbol{n}$ plot.

Which method, out of the three above, does measure the speed of sound (a) most accurate and (b) most precise.

Table: Part 1

Temperature in the room in °C: $T \pm \Delta T =$ ________________

Length of the tube, $L \pm \Delta L =$ ________________ Diameter of the tube, $d \pm \Delta d =$ ________________

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
order of mode = n								
f_n (Hz)								

Table: Part 2

All the node-antinode distances, N-A, are in meters

n	f_n (Hz)	**N-A$_1$**	**N-A$_2$**	**N-A$_3$**	**N-A$_4$**	**N-A$_5$**	**N-A$_6$**	**N-A$_7$**	**N-A$_8$**	**N-A$_{avg}$**
1										
2										
3										
4										
5										
6										
7										
8										

Draw the standing wave patterns for the **pressure** wave for at least 5 resonant frequencies.
Each dotted box has length of L/24

Table: Part 3

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
order of mode = n										
f_n (Hz)										

	11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
order of mode = n										
f_n (Hz)										

Appendix A

1. Finding the Fourier transform of a white noise using Mathematica

Let us describe a white noise (a signal with all the frequencies within a certain range, all with the same amplitude) as a combination of 700 cosine functions of a form **cos(it)**, where **i** goes from 1 to 700, all with an amplitude of 1. The display of couple of functions (cos(t), cos(5t), cos(10t), cos(100t), and cos(700t)) are displayed in figure 1. You can imagine that displaying all of them would fill the figure so wouldn't be any free space. The microphone experiences the signal equal to sum of all, shown in figure 2. It shows sum of 700 cos(nt) functions ($\boldsymbol{n = 1, ..., 700}$) for time $\boldsymbol{-0.1 \leq t \leq 0.1}$. The figure is drawn using expression in Mathematica:

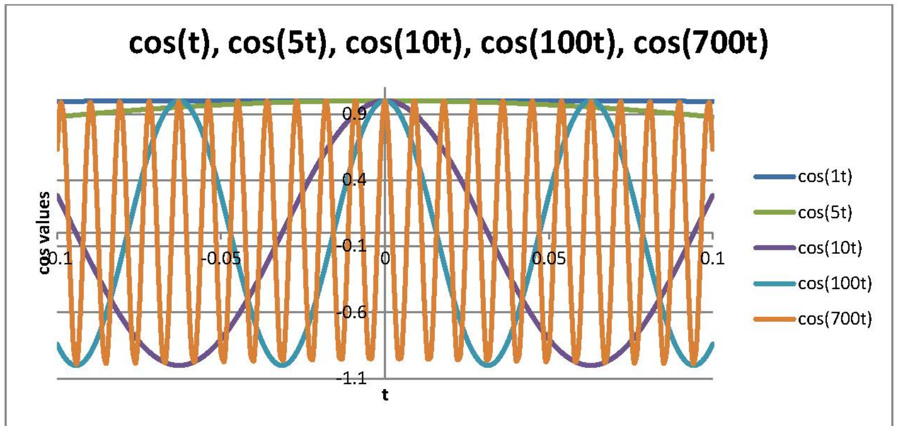


Figure 1. Part of white noise signal

Plot $[\boldsymbol{\sum_{i=1}^{700}(\mathrm{Cos}[\mathrm{it}]), \{\mathrm{t}, -0.1, 0.1\}, \mathrm{PlotRange} \rightarrow \mathrm{All}}]$

In order to prepare the miciriophone signal for Fourier transform we should sample the signal by digitizing the function drawn in figure 2. That produces the table of values of the sum signal for time points 0.001 s apart, from -0.1to 0.1 s, called ftable in the expression used in Mathematica:

ftable = Table $[\boldsymbol{\sum_{i=1}^{700}(\mathrm{Cos}[\mathrm{it}]), \{\mathrm{t}, -0.1, 0.1, 0.001\}};$

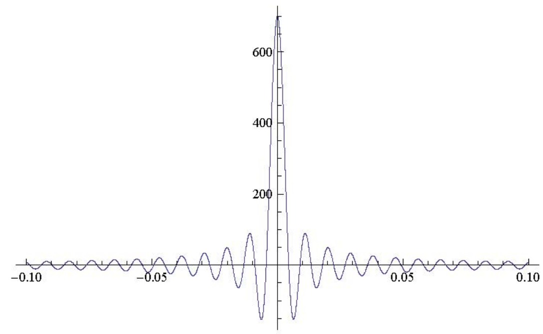


Figure 2.

To check if the table was done correctly one can convert points from the table to the function again, using expression in Mathematica:

ListPlot[ftable, Joined $\rightarrow$ True, PlotRange $\rightarrow$ All]

the plot of which is shown in figure 3.

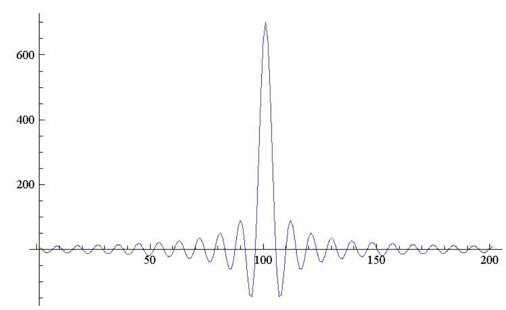


Figure 3.

Table correctly represents microphone signal, so we are ready to use Fourier transform, by using expression in Mathematica:

ListPlot[Abs[Fourier[ftable]], PlotRange $\rightarrow$ True, Joined $\rightarrow$ True]

the plot of which is shown at the left in figure 4. Because of the symmetry of the Fourier transform output, half of the plot is displayed as the Fourier transform of the function, like one shown in the right side of figure 4. That plot has approximatelly constant amplitude, as do the original cosine functions, as we should expect. To check the frequemcies, we can calculate the maximum frequency, of cos(700t) function. The original maximum frequency is $\boldsymbol{f_{700} = \frac{700}{2\pi} = 111.4}$ Hz. Reading from the Forier transform plot, the uncalibrated maximum frequency is about $\boldsymbol{f'_{100} = 23 \pm 1}$ Hz. The actual frequency can be found

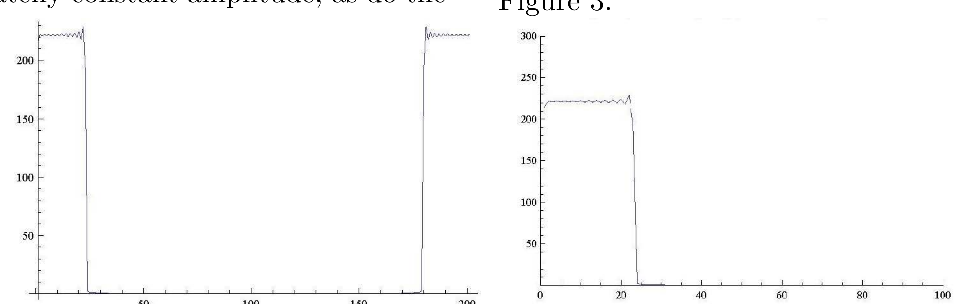


Figure 4.

using the factor $\frac{\mathbf{1}}{\mathbf{0.2}} = \mathbf{5}$ because the time of the white noise signal and sampling is 0.2 s. Therefore, $\boldsymbol{f_{700}} = \frac{\boldsymbol{f'_{700}}}{\boldsymbol{\Delta t}} = \frac{\mathbf{23}}{\mathbf{0.2}} = \mathbf{115}$ Hz and its uncertainty is $\boldsymbol{\sigma_{f_{700}}} = \frac{\boldsymbol{\sigma'_{f_{700}}}}{\boldsymbol{\Delta t}} = \mathbf{5}$ Hz. This result, (115 ± 5) Hz is consistent with the original frequency of 111.4 Hz within one standard deviation.

The last we can check, is the actual Fourier transform. We can perform inverse operation to see if we can get back the initial function (microphone signal). The expression in Mathematica is:

ListPlot[InverseFourier[Fourier[ftable]], PlotRange → All, Joined → True]

The plot of this operation is presented in figure 5. As we can see, the operation brought the original function back.

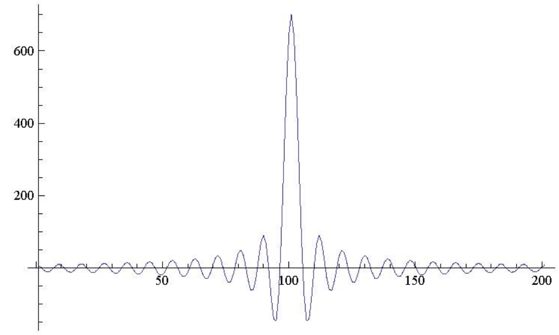


Figure 5.

Appenix B

Describing standing waves using Mathematica

The expression (1) in Mathematica, represents a traveling wave moving leftward (in $-\boldsymbol{x}$ direction), rightward (in $+\boldsymbol{x}$ direction), and the standing wave resulted from the sum of the two waves.

Animate[Plot**{** [Cos[1.5 x - 6 t], Cos[1.5 x + 6 t], [Cos[1.5 x - 6 t] + Cos[1.5 x + 6 t]**}**, **{** x, 0, 2 Pi **}**, PlotRange → 2, PlotStyle **{** Dotted, Dashed, Thick **}**], **{** t, 0, 5 **}**, AnimationRunning → False] (1)

A snapshot of the plot with animation of the expression (1) is shown in figure 1.

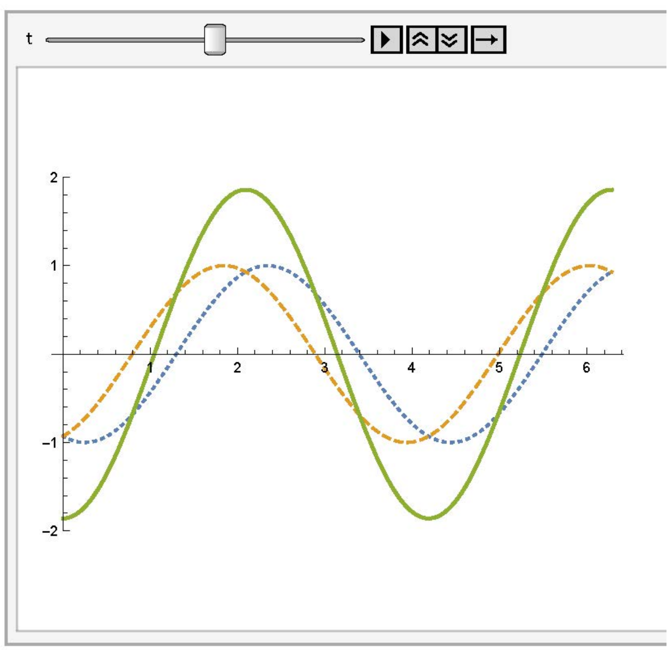


Figure 1. The dotted line represents the wave moving rightward, dashed line represents the wave moving leftward, and the thick line represents the standing wave of the third harmonic.

NOTE: The speed of the waves moving leftward and rightward is the same. The range of the plot represents the length of the resonant tube. The wavelength of the third harmonic is 1.5 times shoter than the length of the resonant tube.