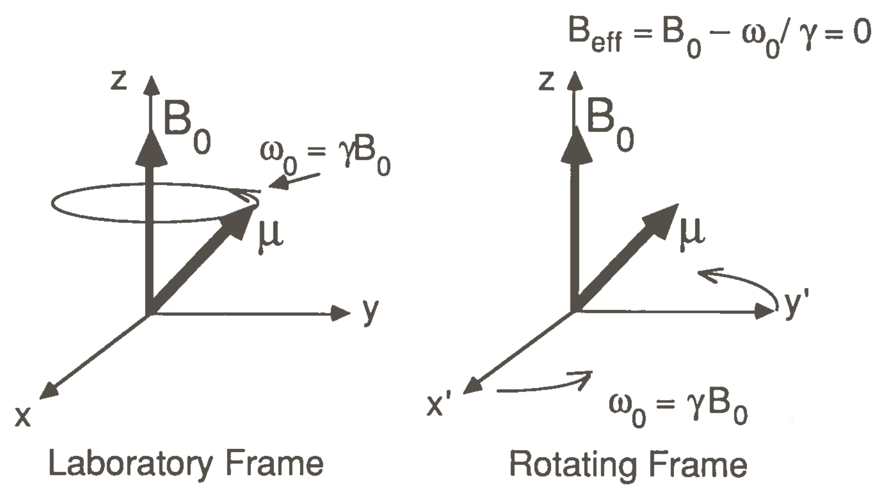

Figure 8.1 Larmor precession of a magnetic moment, μ, about a static magnetic field, $\boldsymbol{B}_0$, oriented along the laboratory-frame z-axis. Left: Magnetic moment rotating at the Larmor frequency about $\boldsymbol{B}_0$ in a static laboratory coordinate frame (x, y, z). Right: Static magnetic moment (i.e., as if no $\boldsymbol{B}_0$ field were present) in a coordinate frame (x', y', z) rotating about the laboratory-frame z-axis at the Larmor frequency.

$$\frac{d\mu}{dt} = \omega_0 \times \mu \tag{8.4}$$

Combining Eqs. 8.3 and 8.4, we find that

$$\boxed{\omega_0 = -\gamma \boldsymbol{B}_0 \quad (\text{rad s}^{-1})} \tag{8.5a}$$

or $$\boxed{\nu_0 = \frac{|\omega_0|}{2\pi} = \frac{\gamma |\boldsymbol{B}_0|}{2\pi} \quad (\text{Hz})} = \text{Larmor (precession) frequency} \tag{8.5b}$$

An extremely useful alternative representation of Larmor precession (see Figure 8.1) is a coordinate frame whose x-y plane rotates at the Larmor frequency about the $\boldsymbol{B}_0$ axis (i.e., z-axis). Just as one rider on a merry-go-round appears stationary to another rider on the same merry-go-round, the magnetic moment, μ, is fixed in direction in a coordinate frame rotating at the same angular velocity as μ. However, because the laws of physics must be preserved in any coordinate frame, we conclude that the applied static magnetic field must be zero in the rotating frame—otherwise, the magnetic moment would be forced to precess about the static field. The great value of the famous "rotating frame" representation is that the rapid motion of μ about $\boldsymbol{B}_0$ can be eliminated, thereby vastly simplifying the analysis of further μ motions induced by resonant oscillating magnetic fields (see below).

More generally, the *effective* magnetic field, $\boldsymbol{B}_{\text{eff}}$, in a coordinate frame rotating at frequency, ω, about $\boldsymbol{B}_0$ is given by

$$\boldsymbol{B}_{\text{eff}} = \left(B_0 - \frac{\omega}{\gamma}\right)\mathbf{k} \tag{8.6}$$

Finally, the *energy* of a classical bar magnet of magnetic moment, μ, oriented at (polar) angle, θ, with respect to a static magnetic field, $\boldsymbol{B}_0$, is given by the dot product (see Figure 8.2):

$$\text{Energy} = -\mu \cdot \boldsymbol{B}_0 = -|\mu|\, B_0 \cos\theta. \tag{8.7}$$

Thus, the magnetic energy is a maximum or minimum when the magnetic moment is aligned against ($\theta = 180°$) or along ($\theta = 0°$) the applied magnetic field. Alternatively, the magnetic energy is zero when μ is perpendicular ($\theta = 90°$) to $\boldsymbol{B}_0$, so that a magnet may be rotated by arbitrary (azimuthal) angle, ϕ, in the x-y plane without changing its energy. We shall return to these properties when we have considered the differences between classical and quantum mechanical magnetic moments.

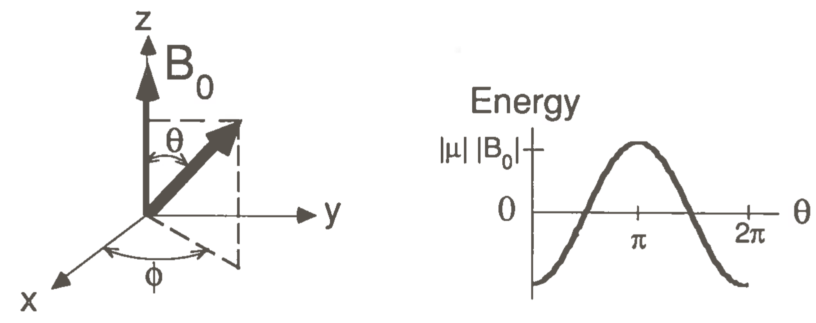

Figure 8.2 Energy of a classical mechanical magnetic moment, μ, as a function of (polar) angle, θ, between μ and an applied magnetic field, $\boldsymbol{B}_0 = B_0\,\mathbf{k}$. Note that the energy is independent of the (azimuthal) angle, ϕ, subtended by μ in the x-y plane.

8.1.2. <u>Quantum mechanical magnetic nuclei</u>

In classical electromagnetic theory, a rotating charge generates a magnetic field whose magnitude is proportional to the rate of rotation (more precisely, the angular momentum) of the rotating charge. Since several atomic *nuclei* (e.g., ^{1}H, ^{13}C, ^{31}P, etc.) have "spin" angular momentum, $\boldsymbol{I}_n$, and are positively charged, they exhibit a *nuclear* magnetic (dipole) moment, μ_n, proportional to $\boldsymbol{I}_n$ as in the classical mechanical case:

$$\mu_n = \gamma_n \boldsymbol{I}_n \tag{8.8}$$

with proportionality constant, γ_n, the magnetogyric ratio characteristic of that nucleus.